PhD thesis proposal
Certified optimization for automated design of reliable digital filters and control systems

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Context

Digital filters and state estimators are essential components of everyday electronics like radios, mobile phones, etc. but also in robotics and aeronautics. Infinite Impulse Response (IIR) filters [1] is a class of recursive linear time-invariant systems which can be also seen as a State-Space (SS) model of a controller. These controllers are basic bricks for any linear dynamic system. In order to improve the accuracy (and efficiency), the state information can be estimated rather than obtained from a real measurement. The most common state estimator algorithms are Kalman Filters [2]. These numerical algorithms for state estimation in dynamical systems have many applications, e.g. in positional control, fault-detection systems and Model-Predictive Control (MPC).

Filters design is a core topic in digital signal processing and control theory, one that has received significant research interest for the better part of the last half century. A lot of effort has gone into constructing flexible filter design methods for both software and hardware. For designing software-based filters with floating-point coefficients, there are many powerful approaches that are relatively easy to use by the filter designer. In the hardware generation space, the situation is much less attractive. There are several reasons for this discrepancy between the software and hardware spaces: algorithms cannot be transferred directly from software to hardware efficiently; fixed-point arithmetic makes hardware design more complex by offering too many degrees of freedom; lack of freely and/or easily available tools for *certified and optimized* filters.

Hardware-implemented filters are crucial in application areas where performance and/or power and area efficiency are critical constraints, such as in drone controllers, WiFi equipment or many other low power embedded devices like those used in the internet of things (IoT). As digital filters are known to be resource- and power-hungry, it is of utmost importance to have the best possible implementation.

The ultimate goal of this thesis is to develop methodology that, from the high-level specification of a digital or Kalman filter, offers the *most efficient but certifiably accurate hardware implementation* in an automated way, reducing design tuning effort.

The classic filter implementation flow passes through several steps: (i) filter design (or plant design for Kalman filters); (ii) quantization step that optimizes filter parameters to be efficiently used in hardware or software; (iii) implementation, in which a valid hardware/software code is generated using quantized coefficients. While each of the stages can be carried out efficiently or even optimally in some cases, the overall result is typically suboptimal as the steps strongly depend on each other. Existing solutions for linear digital filters typically cover only the combinations of the filter design and quantization [3], or quantization and implementation [4].

An inherent part of any design process is a methodology for *generic* analysis of the impact of finite precision over the implemented system. While recently developed in the context of recursive digital filters [3], it is yet to be adapted for Kalman filters, the difficulty of which lies in the non-linearity and highly-dependent iterative computations.

This thesis aims at optimization of hardware resources and the first issue studied is the realization of the hardware multiplications, which are usually multiplications with constants, for which optimization techniques have been extensively explored. The associated optimization problems [5] allow for a massive reduction of arithmetic resources, time and power. However, this reduction strongly depends on the coefficient values, and therefore on the results of the previous design steps. Combination of all the design steps into one non-linear (or linearized) optimization problem is complex and is typically done on a case-by-case basis. In addition, due to the complicated combinatorial nature of the problems, everything is based on heuristics. **Our goal is also to improve on this by investigating various optimization models and techniques and provide automatic problem generation and optimal certified resolution.**

Objectives and Work plan

The goal of this thesis is to define an end-to-end formalization that embraces all the desiderata of digital linear and Kalman filter design and implementation related to hardware efficiency and also to numerical quality. Towards this goal, the development of a custom open-source solver is envisioned during the project. The objectives can be thus assembled in the following research axes:

- *Impact of finite precision upon Kalman filters.* The objective of the PhD candidate will be to provide generic error analysis for all computations within Kalman filter similarly to [3]. The key challenge of this work is in bounding the propagated error through the non-linear filter, and providing an ultimate guarantee of the stability of the Kalman filter through low-precision computations. The research plan consists in first studying existing approaches for recursive digital filters in state-space representation [3], and extending them for optimal hardware-oriented implementations following the recent work [6]. Further, the complete analysis chain for the state-space systems should be extended to Kalman filters.

- *Modeling of filter design and hardware implementation constraints.* We must formulate constraints for the optimization problems. These constraints are based on the digital filter type, implementation technique, hardware model, and the required numerical quality. These problems will in general lead to nonlinear combinatorial models, making it difficult to handle in practice. One key challenge of this work is the choice of suitable modeling technique (linear, non-linear, MINLP, SAT/SMT, etc).

- *Custom solver for heterogeneous optimization problems.* The objective here is to improve the optimization time. The plan is to pursue custom approaches for the family of optimization problems we target, which will involve a combination of different optimization techniques. The PhD candidate will base her/his work on the classic branch-and-bound approach, enhancing it with domain-specific knowledge coming from signal processing and hardware arithmetic. We also plan on combining techniques coming from both linear and non-linear optimization and performing model separation.

Prerequisites

The successful PhD candidate must have obtained the Master in Computer Science and must have a strong background in one or more of the following areas : computer arithmetic, optimization and operation research, digital signal processing and control. Programming skills are expected and background in computer architecture is beneficial.

References

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